AGAINST "EXCHANGE DEGENERACY": THE PHYSICS BEHIND SYMMETRIZATION

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ABSTRACT. It is often asserted that quantum states for same-type particles must be symmetrized due to so-called "label redundancy," i.e. the assumption that the permutations of labels in direct-product states do not reflect any real physical distinction and thus their permutations constitute an "exchange degeneracy". This assumption is directly challenged by the case of scattering of same-type particles such as electrons, which involves two physically distinct scattering channels respectively described by permutation of the labels. I discuss this counterexample with critical attention to an extant portrayal in the literature that suppresses pertinent physical content.

1. Introduction and Background

This paper addresses a specific aspect of the study of symmetrized states in quantum theory. Symmetrized states have long been known to be empirically necessary for correct predictions involving multi-particle systems of the same type. For example, the two electrons in a Helium atom occupy eigenstates which are sums of direct products of individual particle Hilbert space states in which arbitrary labels of the particles (e.g., 1 and 2) have been permuted. These are termed "singlet states" or "triplet states" depending on the collective angular momentum value. For fermions, these states must be "antisymmetrized" (have opposite phases) while for bosons, these states by be symmetrized (have the same phase). The spin singlet state for electrons is:

$$|\Psi_{singlet}\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\rangle \otimes |\downarrow_2\rangle - |\uparrow_2\rangle \otimes |\downarrow_1\rangle) \tag{1}$$

We should first note that a key issue concerns exactly what constitutes a "multi-particle system", since it is acknowledged that in the real world, particles of the same type often do not exhibit correlations signifying entanglement and thus do not seem to comprise multi-particle systems in the sense of requiring an entangled state such as the above.[1] In other words, it is possible for same-type particles to become unentangled, and this remains an ill-defined process in conventional quantum theory (in view of the measurement problem). While this issue arguably underlies much of the problem cluster associated with symmetrized states, for present purposes we focus on one narrow aspect: specifically, whether there is real redundancy in particle labels and how the answer to that question

[1] An answer to the question of "what constitutes a multi-particle system" for symmetrization purposes is offered in Kastner (2023)

affects the status of the so-called "symmetrization postulate". In short, we find that symmetrization is not something that needs to be postulated as a universal rule in order to avoid redundancies and associated inconsistences, but instead is a procedure demanded by specific physics and that describes specific physics. In fact, no genuine redundancies arise when neglected physics is appropriately taken into account. Moreover, we argue that unnecessary redundancies are actually introduced through treating symmetrization as a universal postulate.

The prevailing way of attempting to justify symmetrization in terms of a universal postulate is exemplified by the following passage in Bigaj (2015) which invokes "exchange degeneracy" or the notion of label redundancy (emphasis added for critical purposes):

> "The textbook way to introduce this postulate is through the concept of exchange degeneracy [Cohen-Tannoudji, C., Diu, B., & Laloe, F. (1977)]. Considering the joint state of two particles of the same type such that one of them occupies state $|u\rangle$ whereas the other one is in a different state $|v\rangle$, **we should observe that the two permuted states $|\mathbf{u}\rangle|\mathbf{v}\rangle$ and $|\mathbf{v}\rangle|\mathbf{u}\rangle$ are empirically indistinguishable**. According to the essentialist approach this indistinguishability comes from the fact that both bi-partite states represent one and the same physical state of affairs. On the other hand, the haecceitist approach admits that there is a difference between the permuted and non-permuted states, but this difference cannot give rise to any observational effects, as haecceities are not empirically accessible. In order to avoid the degeneracy problem, we adopt the symmetrization postulate, which narrows down the admissible states to the symmetric (occupied by bosons) and antisymmetric ones (applicable to fermions)."

We will question the bolded passage in Section 3. For now, we observe that Bigaj advocates what he terms the "essentialist" approach which asserts a true label redundancy, while he mentions also a competing "haecceitist" approach according to which the distinction referred to by the permutation is viewed as not related to specific physics but only to a metaphysical principle of fundamental 'thisness'. In keeping with his view that particle labels are truly redundant, Bigaj has advocated an approach that rejects so-called "factorism", Caulton's term (e.g., Caulton 2014) for the idea that the individual direct product states appearing in symmetrized states are physically meaningful.

In direct contrast to this position, the current work specifically defends the meaningfulness of the direct-product states. (We do not however adopt the term "factorism" to describe this position in view of the term's pejorative associations. For example, Caulton claims that "factorist" states cannot satisfy the classical limit (Caulton 2014, 14). However, that conclusion is based on failure to take into account a well-defined measurement process and is directly contested in Kastner, 2023 on that basis. Dieks and Lubberdink (2022) also reject physical meaningfulness of the direct product states).

We address the above interpretive issues further in Sections 3 and 4. We now turn to a specific counterexample to the assumption of label degeneracy exemplified in the Cohen-Tannoudji *et al* extract quoted above. The example calls into question the basic "ground rules" on both sides of the conventional debate (essentialist vs. haecceitist), both of which deny any specific physical significance associated with the label permutation.

2. The scattering example

Bigaj's “anti-factorism” view is a deflationary approach in that it denies that there is any physical meaning in individual system spaces and their direct products. This approach is exemplied in Bigaj's depictions of scattering processes as involving only label-swapping, when in fact there is physical content that, when represented in a Feynman diagram, would readily distinguish the two types of processes. We now examine these details.

Bigaj considers electron-electron (Möller) scattering but does not provide the associated tree-level Feynman diagrams, so let us provide those first for reference in Figure 1. They are called the “t-channel” and the “u-channel”. The former involves simply connected paths from initial points to final points, while the latter involves path crossing, and the u-channel is also called the “crossing channel” for that reason.

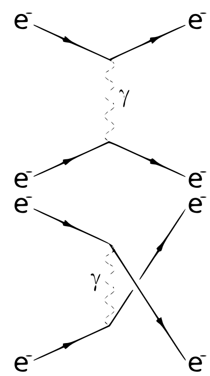

Figure 1. Feynman diagrams for Möller scattering t- and u-channels, top and bottom respectively

We should first note that, contrary to Bigaj's presentation that imposes symmetrization as an *ad hoc* procedure to the incoming and outgoing states (Bigaj 2022, 226), the correct Feynman-diagram analysis of scattering interactions treats all entering and exiting particles as independent in an “appropriate limit” which is usually codified as $t \to \pm\infty$. In the appropriate limit, there is a fact of the matter, empirically grounded, that “electron prepared in momentum state p_1 (or p_2) has spin i and electron detected in momentum state p_3 (or p_4) has spin j”, where i and j are either ‘up’ or ‘down’ with respect to a particular chosen axis. In other words, upon preparation and detection the two electrons are not entangled and will not exhibit EPR-type correlations. This is demanded by the description of the entering and exiting electrons by the free Hamiltonian and the assumption of well-defined incoming and outgoing single-particle states, which is manifest in the relevant Feynman diagrams. (While this is relevant to the issue of what constitutes a “multiparticle system”, we leave discussion of that issue to the specific proposal in Kastner (2023).)[2] The point

[2]This assumption of the independence of the incoming and outgoing electrons, despite being outinely empirically corroborated, cannot be upheld if the evolution is strictly unitary, since it is well-known that EPR correlations are not limited by time or distance. Thus, there remains a lacuna in the conventional

of this observation is to underscore the fact that entanglement warranting symmetrization is not present universally, but arises due to interaction Hamiltonians. While the exchange interaction is often not made explicit, that is what gives rise to two interfering scattering channels. It is described by the exchange integral (cf. Hutem and Boonchui, 2012):

$$K_{1,2} = \int_{\mathbf{r_1}} \int_{\mathbf{r_2}} \psi_a^*(\mathbf{r_1})\psi_b^*(\mathbf{r_2}) \frac{e^2}{|\mathbf{r_2} - \mathbf{r_1}|} \psi_a(\mathbf{r_2})\psi_b(\mathbf{r_1}) d\mathbf{r_1} d\mathbf{r_2} \tag{2}$$

for $\{a, b\}$ designating two different possible spatial states for the particles and $\{1, 2\}$ two arbitrary labels for the particles. Thus *it is the exchange interaction that imposes no fact of the matter about which electron is in which state (a or b), not a formal postulate.* The “indistinguishability” manifesting in (2) arises from the fact that electrons are all excitations of the same quantum field, whose operators are destroying the incoming electrons and creating the outgoing ones. The field does not account for “which electron is in state a and which electron is in state b”. But the entanglement meriting symmetrization arises due to the interaction (and not without its operation). At no time was it necessary to impose symmetrization as a universal rule or invoke a redundancy with respect to permutation. As we shall now see with respect to the details of the channels, in fact there is no such redundancy.

First, we note that Bigaj’s “center of mass” depiction of the two tree-level processes obscures the fact that the u-channel involves crossing, as can be seen in a reproduction of his Figure 8.1 (our Figure 2).

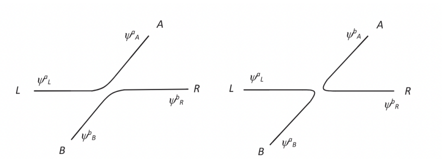

Figure 2. Bigaj’s Figure 8.1

Bigaj first uses his non-intersecting“scattering” depiction (our Figure 2) in reference to distinguishable particles, but then discusses particles of the same type (“indistinguishable particles”) with reference to the same figure. In Figure 3, we focus on the u-channel as sketched in his left-hand diagram. In Bigaj’s depiction, any lab-frame longitudinal propagation is suppressed, as is temporal dependence. This obscures significant physical content, which we now consider with reference to Figure 4.

The depicted R and L electrons wavefunction propagation can be readily seen as a function of time if we parameterize the z (longitudinal) component by $z(t) = t$. (The propagation may not necessarily have a z component, but this helps to visualize the t-dependence, as in a Feynman-type diagram). Then Bigaj’s depiction corresponds to the

theory concerning the conditions for preparation and detection establishing the assumed asymptotic independence of the electrons. This lacuna is arguably remedied in the transactional formulation which has genuine physical non-unitarity under specified conditions; cf. Kastner (2022).

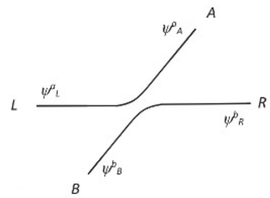

FIGURE 3. Bigaj's Figure 8.1, first image.

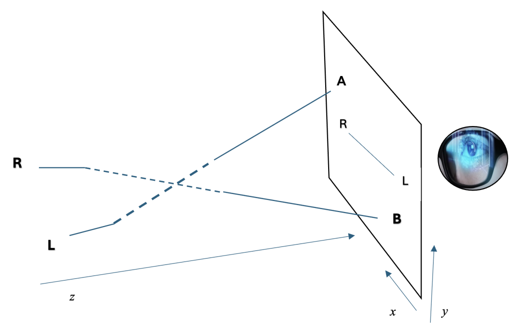

FIGURE 4. Bigaj's depiction from a different viewpoint.

viewpoint of the indicated observer shown here in Figure 4; the observer does not 'see' the z-propagation.

In Bigaj's first diagram, Figure 3, the electrons are shown as propagating towards one another in the x,z plane and then veering off in the plus and minus y directions respectively, never intersecting. (From our viewpoint looking down at Figure 4, the L trajectory is in front of the R trajectory). But in fact, this is a "crossing" or u-channel process that does not exclude the trajectories from intersecting; that is why it is called a "crossing" channel. In particular, if the beginning and ending points are all in the same plane, then the paths must cross. Specifically, let A and B be at the same y-coordinates as the initial states R and L (as illustrated by their indicated projections on the "detection" plane in Figure 4). Then the trajectories would have to intersect when illustrated in the same manner as Bigaj's figure 8.1; i.e., there could be no veering off to different y-values. This is essentially what is depicted in the Feynman diagram for the u-channel. Furthermore, if the propagation had no z-component (i.e., contained in the xy plane), for endpoints A and B at the same y-coordinate, in the u-channel the particle paths would intersect identically throughout.[3]

[3]In using the terminology "path" or "trajectory" we keep in mind that these are *possible* paths, as in Feynman sum-over-paths. These are not determinate classical trajectories. Nevertheless, they are individually physically well-defined in terms of their respective momenta. We might also observe that while one could deny that either trajectory is really continuous and thus evade the intersecting aspect of the

In summary, the t-channel and the u-channel have distinct names because they are topologically distinct physical processes. The two cases depicted by Bigaj as representing nothing more than a swapping of labels are in fact physically different situations, even if each situation individually is empirically inaccessible. This demonstrates that the permuting of the labels is not a mere descriptive redundancy of the same situation. The only way to deny this is to appeal to a full-blown antirealism by asserting that neither the t-channel nor the u-channel qualify as processes or situations; but in that case, neither direct product ket (in Bigaj's notation, $|\psi_A\rangle_1 \otimes |\psi_B\rangle_2$ nor $|\psi_B\rangle_1 \otimes |\psi_A\rangle_2$) refers to a situation. If neither ket refers to a situation, then neither does their sum (i.e., a symmetrized state). On that view (for example) the singlet state of electrons in a Helium atom does not refer to anything. This is a reductio of such an attempt to deny that the direct product states refer in order retain the conventional claim of redundancy or exchange degeneracy. In effect, the claim becomes vacuous at best, since of course quantities that all refer to nothing are "degenerate," but only in a trivial sense.

It is worth emphasizing that the antisymmetry of the fermion states results from the swapping of particle labels required to take into account the two interaction channels. Based on the Feynman diagrams, these channels are first order transition amplitudes from initial direct product states to the final direct product states. The swapping of indices needed to take into account each transition amplitude (i.e. the t- and u-channels) introduces a minus sign based on the anticommutation relations of the field operators. Thus, symmetrization really applies not to a particular state, but to the total transition amplitude from the initial direct product states (typically, specific momentum eigenstates) to the final direct product states, and it is actually a kind of shorthand to apply symmetrization directly to the states themselves.

To clarify, the arbitrary labels appearing in symmetrized states function as surrogates for specific measurement outcomes such as "momenta p_i and p_j" that are actually detected. The latter apply to the outgoing electrons in scattering, where each outcome could have come from either incoming state such that those amplitudes add. For the case of bound electrons such as in a Helium atom in a particular eigenstate of total momentum (singlet or triplet), no outgoing state is yet specified since the electrons remain bound, but the potential for measurement outcomes is reflected in the typical wavefunction representation, where the indices are imported into the position basis; e.g., $\Psi(x_1, x_2)$. Wavefunctions are amplitudes, reflecting the above point. Were a position measurement actually conducted, yielding outcomes such as (A,B), we would then have an outgoing direct product state–not a symmetrized state, just as in the free particle case above. Recall that the outgoing free particles are not entangled and are correctly represented by a product state; this is explicit in the Feynman diagrams. Since they are not entangled, there is no need for the arbitrary labels. Instead they are distinguished by their measurement outcomes. (One might note that this is actually harmonious with Bigaj's criterion for distinguishability; e.g., (Bigaj, 2022, 191). All he need do is take into account that the arbitrary labels need

topological distinction, this does not nullify the observation that each permutation describes a physically distinct type of connection between incoming and outgoing momenta.

not be viewed as permanent tags requiring ongoing formal symmetrization, but instead surrogates for specific measurement outcomes.)

The tradition of attributing symmetrization to states as opposed to amplitudes obscures the essential physics of the exchange interaction, in which symmetrization really applies to the total transition amplitude. This point underscores the fact that the conventional treatment of symmetrization as a universal postulate needed to overcome an alleged degeneracy of direct product states is detached from the actual physics of the interactions taking place, which in general concerns transition amplitudes and which (in the real world) goes beyond first-order. If one were to take into account higher order effects, there would be appropriate additional contributions not limited to conventional symmetrization (e.g. Banerjee *et al* , 2022)).

Another observation is in order regarding the crucial role of the exchange interaction, which has its source in the Coulomb force as well as in the fact that one is dealing with quanta of the same field. A typical discussion of the exchange interaction (e.g. as in Hutem and Boonchi 2012) starts with a symmetrized Helium eigenstate and evaluates the expectation value of the Coulomb perturbation with respect to that state. But the interaction is general and also applies to a situation involving free electrons in a direct product state; it is the interaction that couples those states, and that is what is expressed in the above scattering situation and in (2). The latter can be more directly seen from the application to ferromagnetism in Jeschke (2018).

3. "Redundancy" Orthodoxy Neglects Crucial Physics

The above counterexample demonstrates that both the approaches mentioned above–essentialism and the traditional haecceitistic approach–neglect crucial physics that constitutes a meaningful distinction between the permuted states (even if not an empirically accessible one). Thus, arguably there is in fact no redundancy, and accordingly no real degeneracy problem that needs to be avoided or remedied through the postulation of symmetrization as a universal formal rule. Instead, the summands in symmetrized states are indeed individually physically meaningful; specifically, they refer to distinct processes that must be combined in the appropriate phase. This points to a need to critically evaluate the implicit assumed equivalence between ontology and 'observational effects', as evidenced in this comment from the above extract from Bigaj (2015): "but this difference cannot give rise to any observational effects, as haecceities are not empirically accessible." Here, Bigaj presupposes, without argument, that any physically relevant difference must be one that "gives rise to observational effects." That is, in the conventional debate on both sides, *the physical relevance of any ket (including a direct product ket) is tacitly assumed to require that it be independently associated with specific observational effects.* This assumption is directly challenged by the scattering counterexample presented above: the processes described by the permuted kets are physically distinct, and both are evidently necessary—in the appropriate phases—for ultimately correct empirical correspondence.

Thus, the current work appeals for a critical awareness of the underlying, largely unexamined empiricist assumption underlying the conventional debate–specifically, that only that which directly corresponds to an observational effect should be considered physically real or at least physically relevant–which persists on both sides of the current debate between what Bigaj refers to as "orthodoxy" versus his own"heresy". (The "orthodox" camp in Bigaj's formulation is exemplified by the work of French and collaborators; e.g., Krause and French (2007).)

In any case, in reference to the boldfaced passage quoted above, one cannot term the permuted states "empirically indistinguishable" since in fact (for entangled systems) such states are empirically *inaccessible*, and these two terms are not equivalent (Kastner 2023). Specifically, if one has no empirical access to either situation A or situation B, such that one has associated empirical phenomena for neither A nor B , then when cannot say that these situations are empirically indistinguishable. For the latter implies that A and B instantiate the same empirical phenomena, which is not the case, since they instantiate *no* empirical phenomena (for entangled systems). Despite that fact, the conflation of "empirically inaccessible" with "empirically indistinguishable" seems to have become standard practice in the conventional debate (and arguably this has led it astray).

With this background, let us take a closer look at the notion of "factorism" mentioned in Section 1. Bigaj defines "factorism" in terms of the assertion that the individual Hilbert spaces appearing in direct products "represent states and properties of one individual particle" (Bigaj 2022, 32), but the dependence of this definition on the notion of "properties" makes it somewhat ill-defined. For example, one cannot necessarily identify such 'properties' with eigenstates of observables; but to view them as "beables" is not necessarily justified either. From the context, however, it seems clear that Bigaj and Caulton reject any specific physical meaningfulness of non-symmetrized direct product states. Bigaj seems to allow some vague room for physical meaningfulness in certain comments such as "Operators which represent properties of individual particles are meaningful but, strangely enough, they are not literally observables." (Bigaj 2015, 60). As noted in Kastner (2023), this only seems "strange" under the empiricist assumption that ontology should be equivalent to observability. This slide is further evidenced in Bigaj's argumentation that the direct product states cannot actually be "occupied" by systems, for example in Bigaj (2022,55) where he says:

> The quantum case of permutation-based redundancy is different from the classical one...not only are the kets $|\phi\rangle_1|\psi\rangle_2$ and $|\psi\rangle_1|\phi\rangle_2$ distinct; they are also orthogonal, which means—according to...the Born rule—that the probability of finding the system in one state given that it occupies the other one should be zero. So it seems that the inclusion of both kets in our representational framework leads to a logical contradiction.

However, a contradiction only arises under Bigaj's presupposition that such states should represent the same physical situation. We see in the scattering example that this is simply false.

It may also be noted that the scattering interaction is a direct counterexample to Caulton's "second reason" for denying the meaningfulness of the direct product states. He says:

> My second criticism of factorism is that it defies an interpretative principle that ought to be compulsory; namely that the unitary equivalence of two Hilbert spaces and accompanying algebras is a sufficient condition for considering those Hilbert spaces to be equally good mathematical representations of the same space of physical possibilities. (Caulton 2014, 14)

While it is encouraging to see Caulton embracing the notion of physical possibilities as inherently meaningful, it appears that his mistaken conclusion of redundancy of these possibilities arises from limiting his analysis to the non-interacting Hilbert space. In other words, the algebra of a non-interacting space does not exhaust the relevant physics. To assume that it does amounts to neglecting the exchange interaction, as reminded above. The latter is what results in the need to take into account the above distinct scattering processes and arguably symmetrization in the first place, as implied by (2).

4. Uncritical empiricism needlessly constrains the space of interpretive solutions

The uncritical equating of "empirical" to "physical" and "meaningful"—i.e., the idea that nothing can be considered physically meaningful unless it is directly associated with an empirical phenomenon—can also be seen in Bigaj's elaboration on the "argument from exchange degeneracy" as follows (emphases added):

> Suppose that indeed it is possible for a two-element system of same-type particles to occupy states that are neither symmetric nor antisymmetric, in particular states that are products of two non- identical (orthogonal) vectors. Let us select one such state of the form $|\phi\rangle_1|\psi\rangle_2$. As we already know, the Hilbert space $H \otimes H$ also contains a permuted vector$|\psi\rangle_1|\phi\rangle_2$ which, **by assumption, represents a state that is empirically indistinguishable from** $|\phi\rangle_1|\psi\rangle_2$. Thus we have a case of what is known as representational redundancy: our mathematical framework contains distinct representations of **the same physical, or empirical,** situation. (Bigaj 2022, 54)

However, again, nobody really is (or at least nobody should be) *assuming* that the two states mentioned above are "empirically indistinguishable" since (when interference effects are present) they are never individually instantiated. As noted above, the most that can be said is that they are empirically inaccessible. But it does not follow that these states are not physically meaningful as component states, contrary to the slide in the last quoted sentence that explicitly (and again, uncritically) equates "physical" to "empirical." In contradiction to this assumption, we have seen that such component states have distinct well-defined physical referents in connection with the above scattering example.

Another consideration directly contradicts the idea that the direct product state permutations are "empirically indistinguishable": If expressions $\psi_A(x_1)\psi_B(x_2)$ and $\psi_B(x_1)\psi_A(x_2)$ individually lead to different probabilities, which they certainly can, then they are empirically distinguishable, at least in principle. That is, *if* these states were instantiated by real systems, we would find them empirically distinguished. This recalls the point in Kastner (2023) that one standard argument for the need for symmetrization is that the direct product states taken individually are *noninvariant* concerning empirical phenomena and thereby empirically distinguishable in principle. (Oddly, Bigaj acknowledges that the individual direct product kets are non-invariant, yet still falls in with the convention that these kets are "empirically indistinguishable." He says: In particular we can't distinguish two possible final states after detection $|\psi_A\psi_B\rangle$ and $|\psi_B\psi_A\rangle$, since these kets are not permutation-invariant." Bigaj 2022, 241. But why would we *not* be able to distinguish two things that are non-invariant under permutation? Isn't this exactly the opposite of what is meant by non-invariance–i.e., that the two situations are distinct? Apparently, what he means instead is just that the non-invariant final states are never actually observed. But again, that just means that they are individually empirically inaccessible.)

Besides the uncritical equating of "physical" to "empirical", Bigaj's assertion that "both [direct product] kets are supposed to represent the same empirical situation" (Bigaj 2022, 55) cries out for critical examination, especially in view of the above point that their absolute squares can be different. Whence such an expectation? As far as this author is aware, there is no such edict or requirement. The locution "supposed to represent the same empirical situation" simply expresses the unexamined empiricist assumption that there can be no distinction between the kets because they are empirically inaccessible and therefore cannot individually refer to anything physically real (which we have disputed herein).

Thus, a form of uncritical empiricism has crept into the discussion that serves to preemptively exclude the possibility that the named direct-product states physically refer. This exclusion constrains the study of symmetrized states in a way that arguably has led it off track. Basically, the prevailing orthodoxy has us assigning symmetrized states to same-type particles universally without real physical justification, thereby (besides resulting in vexing interpretational puzzles) introducing a real and arguably unnecessary redundancy in the formalism, to which we now direct our attention. This issue can be introduced by way of Bigaj's stated adherence to the prevailing orthodoxy despite his acknowledgment of its inconsistency with the empirical facts:

> "More precisely, all fermionic states are formally entangled, since no antisymmetric state can be written in the form of a product of vectors.... However, this apparent prevalence of entanglement has no support in experimental facts. For instance, we do not observe non-local correlations connecting all electrons in the universe. Thus there is a need to come up with a new concept of entanglement that would be better suited to the task of describing systems of same-type particles. (Bigaj 2022, 153)

Indeed, among experimental facts inconsistent with symmetrized state representations are measurement results such as "the particle detected on the left has spin up and the particle on the right has spin down" where conducting subsequent measurements on that pair will show a lack of entanglement; i.e., their subsequent behavior is *de facto* not described by a symmetrized state such as a singlet or triplet. However, rather than take the experimental facts as a refutation of the idea that symmetrization is a universal postulate, orthodoxy retains symmetrization and looks for ways to come up with symmetrized states that are yet unentangled. This results in real redundancy, as in the 'pseudo-singlet state' in which the particle with spin 'up along z' has location L and the particle with spin 'down along z' has location R:

$$|\Psi_{pseudo-singlet}\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle_1|R_1\rangle \otimes |\downarrow_z\rangle_2|L_2\rangle - |\downarrow_z\rangle_1|L_1\rangle \otimes |\uparrow_z\rangle_2|R_2\rangle) \quad (2)$$

In this construction, symmetrization becomes an artificial procedure dutifully entered into under the orthodox edict that "all same-type particles must always be in symmetrized states". On the contrary, a physically sufficient description would be the simple product state $|\uparrow_z\rangle|L\rangle\otimes|\downarrow_z\rangle|R\rangle$ where it is acknowledged that this result arises from a disentangling measurement. Given that process, it is no longer necessary to use numerical labels for the particle subspaces, since in this case those labels were just surrogates for a possible measurement result, as discussed in Section 2. One cannot respond with "but that direct product state is not allowed," since indeed it is explicitly used, for example in scattering experiments as a description of two prepared incoming or outgoing electrons (recall Section 2). That is, such states are simply free solutions to the Dirac equation, routinely used to describe free electrons such as those entering or exiting a scattering center. This fact points to yet another reason to reject the idea that symmetrization should be a universal postulate; we don't actually need to use it in the lab to correctly account for experimental phenomena. The numerical labels only enter when one needs to take into account the two possible transitions (channels) from the initial to final free states as required by the interaction (3).

Of course "measurement" –capable of yielding definite outcomes such as a well-defined Dirac wavefunction–is a fraught subject in the orthodox approach (and that is an aspect of the problem in accounting for disentanglement of same-type particle). Nevertheless, one cannot even assign such determinate properties as in the "psuedo singlet" to individual particle in the absence of a measurement yielding an outcome (and that is why they are stipulated in Bigaj's proposal, which is a variant of orthodoxy). (In Kastner (2023, 13-14) we mention a well-defined process of measurement that can serve to disentangle same-type particles, along with localization.)

It is worth stressing the point that much of the debate has been premised on an untenable assumption of physical meaninglessness or "redundancy of description" around mathematical objects in quantum theory that may well have important physical content. Another example of this dismissal is Bigaj's assertion that phases of quantum states constitute "surplus structure":

> Representational redundancy is a common occurrence in mathematical physics, thanks in part to the richness and flexibility of mathematical formalism (which implies in the majority of cases the existence of so- called surplus structures, in Redhead's terminology, see Redhead 2002). A well-known case of redundancy present in quantum mechanics is caused by the fact that two vectors that differ by a phase represent the same physical state. (Bigaj 2022, 54)

But in fact, this assertion depends on a very restricted and arguably deficient reading of what counts as a "physical state." The Aharonov-Bohm effect—an empirically verified phenomenon—occurs precisely because of phase changes accumulated by the relevant quantum states, and the influence of forces governed by interaction Hamiltonians are effected by relative phase changes in the components of superpositions. Thus it cannot be claimed based on the fact that, e.g., global phases cancel out in calculating an expectation value, that phases are mere mathematical, descriptive redundancies. The latter is part of an unnecessary and unwarranted deflationary "what you see exhausts what exists" tradition towards the theoretical content relevant to same-type particles and quantum systems in general.

Moreover, Bigaj's analysis (Bigaj 2022, Section 3.2) shows that it is only under the arguably mistaken assumption of permutation invariance (based on "exchange degeneracy") that the universal postulation of symmetrization becomes necessary. Ironically, that section shows that Bigaj's essentialist interpretation of the exchange of indices leads to contradictions, which are only remedied through an *ad hoc* symmetrization postulate (SP). Thus the SP only arises to cure the false problem arising from inappropriately taking the direct product kets as representing the same physical situation. In fact, we clearly see in the correctly rendered scattering situation (Figure 1) that the two distinct kets do not represent the same physical situation, so there is no actual exchange degeneracy, and thus no need to impose symmetrization as an *ad hoc* postulate to cure the contradictions arising from the assumption of exchange degeneracy. Instead, symmetrization is physically required by the need to have both processes included and the fact that neither is more physically important than the other. In this understanding, there is no "surplus structure". The iniividual direct product kets are physically meaningful in describing distinct processes, however empirically inaccessible they might be.

5. Conclusion

It has been argued that symmetrized states of same-type quanta, such as electrons, arise not from "permutation redundancy" but from a specific physical interaction Hamiltonian, where the interaction destroys and creates states of the same quantum field. A specific physical process, Möller scattering, involving the Coulomb interaction, has been showed to be a direct counterexample to the conventional claim that symmetrized states need to be imposed universally based on an alleged redundancy or degeneracy of permuted direct-product states. Rather than "exchange degeneracy," there is a specific interaction demanding an exchange of particle labels corresponding to two distinct physical processes. Thus, the direct product states of individual particle spaces are indeed physically meaningful even if they are not individually observable. The symmetrized sum of states arises because there are two equally important ways that the initial and final states can be connected through the specified interaction.

Moreover, the Feynman diagrams for Möller scattering, as well as many other scattering processes among same-type particles, assumes that incoming and outgoing particles are in well-defined individual Hilbert space states, and predictions based on those calculations are routinely empirically corroborated. This fact, as well as the physically distinct features of the scattering channels, refutes the claim that symmetrization is, or should be, considered universal based on alleged redundancy of the permuted direct product states. It has been noted that it is self-contradictory to assert a redundancy of permuted direct product states when those same states are mutually orthogonal, distinct, and lead to different probabilities. Thus, in fact no *a priori* redundancy exists that needs to be remedied by symmetrization. On the contrary, imposing symmetrization on manifestly non-entangled quanta is what actually creates redundancy. This is the case for the so-called "pseudo-singlet" state in which there is a fact of the matter that one particle is in state "Leftward momentum, spin up" and the other is in the state "Rightward momentum, spin down". Artificially constructing a symmetrized states out of this unentangled state simply adds an unphysical redundancy.

At a more general level, an effort has been made to disambiguate an equivocation in the literature concerning "empirical indistinguishability" of states, where that has been inappropriately attributed to direct product states that are not empirically accessible. It has also been pointed out that much of the existing literature adopts an unexamined empiricist stance in effectively equating ontology to observability. The discussed counterexample shows that situations that may not be individually observable, or lead independently to observable effects, are crucially physically relevant.

6. References

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